ISSN 1980-5411 (On-line version)
Production, 33, e20220053, 2023
DOI: 10.1590/0103-6513.20220053

Research Article

A hybrid novel method to economically evaluate the carbon dioxide emissions in the productive chain of Argentina

Federico Gabriel Camargo[a]*

[a]Universidad Tecnológica Nacional – UTN, Facultad Regional de La Rioja – FRLA y Concepción del Uruguay – FRCU, Electromechanical Department, La Rioja, Argentina

*camargof@frcu.utn.edu.ar

Abstract

Paper aims: This paper compares the economic valuation (Intrinsic Cost or IC) of carbon dioxide emissions (carbon price) of the externality taxes (Kyoto Protocol) of the Argentine Production Chain (APC).

Originality: The experimental IC index and this novel combination allow incorporating objective (incremental) and subjective (acceptance, hierarchy and uncertainty) evaluation of the APC. This method is extended in this paper (to other operators).

Research method: The ICs are applied to attribute graphs in two scenarios (APC and society), that are obtained from the combination of Fuzzy Decision Making (FMD), Analytic Hierarchy Process (AHP) and PSO with the Algebraic (AP), Generic Hamacher (GHP and PHP) and Einstein (EP) products.

Main findings: (i) The coherency of the new ICs is demonstrated by mathematical and graphical analysis; (ii) The most demanding operators are the EP and HP.

Implications for theory and practice: It contributes to establish mechanisms for regulation of the APC.

Keywords

Carbon price (externality taxes) of Argentine Productive Chain (APC). Fuzzy Decision Making (FDM) and Intrinsic Cost (IC) index. Variants of the fuzzy operators and Analytic Hierarchy Process (AHP). Algebraic Product (AP). Generic Hamacher (GHP). Particular Hamacher (PHP) and Einstein (EP) Products.

How to cite this article: Camargo, F. G. (2023). A hybrid novel method to economically evaluate the carbon dioxide emissions in the productive chain of Argentina. *Production*, *33*, e20220053. https://doi.org/10.1590/0103-6513.20220053

Received: May 10, 2022; Accepted: Dec. 9, 2022.

1. Introduction

Today, it is urgent to mitigate the sustained growth of CO_2 emissions in the atmosphere, which causes irreversible climate change. The limited consent for the community is 450 parts per million, which implies a temperature rise of 2ºC above the pre-industrial temperature. Environmental sustainability as the sustained growth of carbon dioxide (CO_2) emissions would lead to irreversible climatic changes. Energy sustainability as the depletion of non-renewable resources would lead to an energy crisis. In this context, the improvement of 'Energy Efficiency' is the one with the greatest impact (Camargo & Schweickardt, 2014; Camargo, 2021, 2022a). The problem associated with carbon dioxide emissions is currently difficult to solve, and the proposals to solve it are still under discussion. Improving energy efficiency is one of the tools proposed to reduce emissions. This includes the improvement of the equipment used by the demand, as well as the improvement of the efficiency of the production chain. The consequence of poor energy efficiency and exponentially growing demand for energy is carbon dioxide emissions. One of the proposals made is the establishment of a carbon market associated with emissions, where emission quotas are established, which can be traded.

This market was established in the Kyoto protocol, where three market mechanisms were built (Maamoun, 2019; Miyamoto & Takeuchi, 2019; Silajdzic & Mehic, 2018):

- The Emissions Trading Mechanism (ETM) allows registered countries to trade emission permits or so-called Assigned Amount Units (AAUs) with other registered countries. If one country's emissions are below its limit then it can trade to another country that has exceeded its limit. The European Union, Canada and Japan are part of this trading system.
- The Joint Implementation Mechanism (JIM) allows the commercialization of emission reductions produced by projects that reduce emissions within registered countries. The Units sold are Emission Reduction Units (ERUs).
- The Clean Development Mechanism (CDM), where project-based emission reductions are negotiated. This is the only mechanism that allows developing countries to voluntarily sell emission reductions to registered countries. The CDM has been designed with two objectives in mind: to contribute to the sustainable development of developing countries and at the same time increase the opportunities for registered countries to meet their Kyoto commitments.

The carbon market is an example of Pigouvian taxes and the Coase theorem (Jeanne & Korinek, 2019; Shahzadi et al., 2020). The Coase theorem states that if property rights are established, and there is maximum efficiency, the transactions are carried out at zero cost. If only one has the property rights of the environment, but its activity has a negative impact on the other, so that only one sector can maximize its benefit on the resource, then Pigouvian taxes are established. Pigou argued that the private sector only seeks to maximize its own (marginal) profit. However, in those cases where the social interest differed from the private interest, the industry had not incentive to internalize the social cost of its actions (marginal). In other words, the "invisible hand" of the market does not produce the expected optimal result, maximizing aggregate welfare.

Today this "market failure" is called an "externality" (Cavallaro et al., 2018; Jeanne & Korinek, 2019; Ichihashi, 2021; Silajdzic & Mehic, 2018). Externalities can be positive when there are individuals who benefit from actions that others perform without having to pay for it. This divergence between private and social interest had two effects. In the first place, if the externality was positive, the sector that receives the social benefit does not pay for it. The sector that causes the damage in a negative externality also does not pay the damage to the injured parties. Second, when the (marginal) social cost was greater than the (marginal) social benefit, the generator of the damage tended to over perform said activity, mainly because it did not face all the associated costs. Then, it was then recommended to implement a tax on the generator of the externality. Previously, taxes caused an imbalance in the economy. In this case, the aim was to solve a social imbalance by imposing a tax. In other words, the Pigouvian tax makes the generator of a negative externality (pollution) pay for the (marginal) damage that it inflicts on the rest of society, which was revolutionary. Then, the state must correctly estimate the private benefit and the social cost of the externality, in order to obtain the tax to apply. This is not easy to calculate in practice.

So, the emission mechanisms seek to establish a tax or subsidy, whose price is established by the laws of supply and demand, based on the transaction of property rights. The determination of that price depends on the upper and lower limits and on the preferences of the participating sectors (Camargo et al., 2018). These data are recurrently obtained from statistical analysis, mathematical models and the use of complex models in operations research (see Table 1).

Table 1 shows a summary of the present proposals of carbon price and CO_2 emissions with: country of analysis, sources analysed, linear or non-linear modeling, type of uncertainty, resolution methodology and limitations. Most of the important published works are carried out using deterministic models, or in the best of cases, mixed with stochastic models. That is, they use classical models of mathematical programming, which have limitations in their formulation and quality of the results obtained (Camargo et al., 2018; Camargo, 2022b). It is observed that these proposals present complex methods, which require a large amount of data, in some cases require supervised learning and most do not consider the presence of uncertainty, but rather are statistical analyses. Some present proposals based on Machine learning (Janiesch et al., 2021) which has the disadvantage of requiring supervised learning. Others present techniques based on the impact on the market equilibrium. Computable general equilibrium (CGE) models are a class of economic models that use actual economic data to estimate how an economy might react to changes in policy, technology or other external factors (Robson et al., 2018; Pradhan & Ghosh, 2022).

According to this review, there are the following aspects that affect the solution of the problem:

Table 1. Search results in the Scopus database (conducted in October 2022).

Author (year)	Country	Uncertainty	Modelling	Indices	Comment
Cavallaro et al. (2018)	Europe	Deterministic	Well-to-Wheel emission	CO_2 emission and Economic value	Analysis from statistical records
Zhang & Zhang (2018)	China	Deterministic	Computable General Equilibrium model	Carbon tax and economic welfare	Analysis from statistical records
Lin & Jia (2019)	China	Deterministic and probabilistic	Computable General Equilibrium model	Carbon tax, energy demand in industry	Analysis from statistical records
Li et al. (2018)	Liaoning (China)	Deterministic	Impacts of regional unbalanced carbon price	Carbon tax and industrial competitiveness	Analysis from statistical records
Simshauser (2018)	Great Britain	Deterministic	Price discrimination and the modes of failure in deregulated retail electricity markets	Carbon price and CO_2 emissions	Analysis from statistical records
Aydin & Esen (2018)	Europe	Deterministic	Dynamic panel threshold regression model	Environmental taxes and CO_2 emissions	Prediction from statistical records
Sun & Huang (2020)	Beijing and Shanghai	Deterministic	Secondary decomposition algorithm and back propagation neural network	Carbon price and CO_2 emissions	Analysis from statistical records
Sun & Zhang (2018)	China and Europe	Deterministic	Multi–resolution singular value decomposition and extreme learning machine	Carbon price and CO_2 emissions	Prediction from statistical records
Wei et al. (2018)	Shanghai and Hubei	Deterministic	Wavelet transform and Kernel-Extreme Learning Machine	Carbon price and CO_2 emissions	Prediction from statistical records
Hao et al. (2020)	China and Europe	Deterministic	Extreme learning machine using a multi-objective algorithm	Carbon price and CO_2 emissions	Optimal prediction from statistical records
Zhao et al. (2021)	Global	Deterministic	Multi-factor decomposition and integration carbon price forecasting model	Carbon price and CO_2 emissions	Prediction from statistical records
Song et al. (2019)	China	Fuzzy -stochastic	Fuzzy stochastic model for carbon price prediction	Carbon price and CO_2 emissions	Prediction from statistical records
Tsao et al. (2021)	Taiwan	Fuzzy-deterministic	Eco-efficient supply chain network and fuzzy optimization	Carbon trade and trade-credit	Prediction from statistical records

Source: The Author.

- There is imperfect knowledge/rationality (Camargo et al., 2018; Camargo et al., 2019; Camargo, 2022b), since the preferences (weights) associated with each affected country and individual are unknown (Cavallaro et al., 2018; Jeanne & Korinek, 2019). Firstly, there is uncertainty about the economic impact of the externality on society and a tax according to the social damage caused by emissions. Secondly, the fact that the marginal value of bonds is tied to the law of supply and demand encourages financial speculation, buying cheap bonds and selling expensive bonds. This volatility introduces uncertainty. Thirdly, elasticities associated with carbon dioxide emissions are unknown. This is because they depend on the economic, political, development, etc. conditions of each country. These conditions are fluctuating; therefore, it is not accurate to perform a ceteris paribus analysis (Maamoun, 2019; Miyamoto & Takeuchi, 2019; Ichihashi, 2021). This partial or total lack of knowledge means that deterministic models (with full certainty in functional relationships) and probabilistic models (with functional relationships based on probability functions) are not suitable. Statistical analysis contemplates the risk according to known situations, but does not contemplate the occurrence of events whose probability of occurrence is unknown (such as economic crises and wars). This uncertainty is not contemplated in almost all the articles analysed.
- Those models that are only deterministic and probabilistic (not fuzzy) do not take into account value uncertainties. This is a problem in a context where there is uncertainty that affects the decision maker. These proposals do

not take into account hierarchy criteria either or, in the best of cases, they make linear weightings. The latter is an inconvenience due to metric compatibility (same measurement units) and the presence of subjectivity and uncertainty in the assessment of each index (Camargo et al., 2018; Camargo et al., 2019; Camargo, 2019, 2021, 2022a, 2022b).

- These methodologies have problems of metric compatibility between indices (Camargo et al., 2018, 2019; Camargo, 2019, 2022b). This is because these indices are required to be translated through a subjective economic cost (a priori). The costs of carbon bonds are associated with their marginal value (a priori), according to the supply and demand transaction. They do not contemplate the particular situation of a given country, market, emission margins and preferences. Objective functions are nonlinear at best, so determining this cost by means of Lagrange multipliers is not efficient. The linearization of these functions makes the solution obtained unsatisfactory in terms of cost, quality and social impact.
- Then, the assessment of the externality of the productive chain is subjective, since the impact (negative for emissions) cannot be fully translated into an economic value (Camargo et al., 2018). This is because the economic cost of environmental and health remediation must be considered, an economic pay for social damages. The preferences and subjectivities of the different countries involved, as well as the consumers and producers of each country, are not considered. The externality implies an impact (in this case negative), which must be measured within regulatory limits, in this case CO_2 emissions. These limits must take into account the situation of each of the parties involved, in addition to their preferences. In the case of the carbon market, the agreed price does not contemplate the development of the countries involved, with developing countries being harmed and developed countries benefiting. The established price is based on the total supply and demand for carbon bonds, without considering the situation of each particular market (Cavallaro et al., 2018; Jeanne & Korinek, 2019; Ichihashi, 2021).
- Formulating these problems requires expensive and complex software. The complexity is due to methodological gaps and disagreement in the definition of indices and evaluation criteria, since some of them are contrary (investment cost vs. CO_2 emissions). Then, there is no agreement on the most appropriate regulatory mechanism to evaluate the penalties (tax or Carbon price) required to guarantee the lower quality criteria required. This is because these attributes are not subject to the conventional laws of market equilibrium and, consequently, are not directly and objectively monetizable. In the case of emissions, attempts were made to establish carbon credit markets, which did not achieve the desired success (Camargo et al., 2018, 2019; Camargo, 2019, 2021, 2022a, b).

Then, three aspects stand out: the fundamental uncertainty is not considered in the state of the art, difficulties in metric compatibility and the presence of non-linearity. For these reasons, the determination of this cost (Carbon price) is not efficient using the classical methods of mathematical programming and statistical analysis.

This work identifies the advantages and disadvantages of the variants of a novel index that measures economic valuation (IC) of the CO_2 emissions of the Argentine production chain. The IC is from the determination of the upper and lower limits allowed, as well as the preferences of the decision maker. The IC index was determined by the present line of research on the reliability of energy systems using the Einstein Product. In this work, novel incentive and penalty mechanisms are sought to improve energy efficiency and improve the carbon bond exchange market, based on this premise. To do this, it was determined the convenience of the tools compared in this article. This proposal is original and has not been published in other articles (with this diversity of t-norms and this problem). A first model of this methodology was applied to the reliability of power systems, where only the Einstein Product was used (Camargo et al., 2018).

The way of obtaining the index used in this article, which was improved by the present line of research, allowed obtaining the index for other t-norms (Algebraic, Hamacher and Einstein). Fuzzy models contemplate the reasoning and perception of the human being and allow change of domain to the studied variables or functions. In this way, there are degrees of acceptance of a certain variable to a given set. The exponential weights are associated with the decision maker preferences (Productive Chain and Society perspective) and its hierarchy criteria obtained by the Analytic Hierarchy Process (AHP) method and Perron's eigenvalues (Section 2.1).

The contributions of the present work are the following:

- The uncertainty (called fundamental uncertainty) allows the problem to be modelled using possibilistic models (fuzzy models). An acceptance level associated with each index is introduced according to the preferences determined by the decision maker (Shahzadi et al., 2020; Camargo, 2019, 2022b). The presence of fundamental uncertainty is contemplated, it is associated with the partial/total ignorance of this problem, due to the factors mentioned (ignorance of functional relationships of the impact of the externalities of CO_2 emissions, the elasticities of demand, social impact and equity).

- The definition and the intrinsic cost graph (Camargo et al., 2018) is extended for different types of t-norms, which was not possible before the improvement made in this index. The rigor of this graph is evaluated based on the confluence operators, to determine the advantages and disadvantages of each t-norm. An evaluation of the investment cost and CO_2 emissions of the Argentine production chain for different combinations of the two attributes is carried out and represented in a two-dimensional graph. The intrinsic cost (marginal cost) is obtained post-optimization (a posteriori) through novel indices obtained from fuzzy decision theory (possibility models) and AHP (preferences) is used. These indices consider the economic cost associated with worsening or improving the index in question. In this way, a contribution is made to fuzzy decision theory by contributing a novel index to the state of the art. In this sense, different variants of fuzzy relations and level of preferences are used to analyse the effect produced.
- The preferences of the decision maker are contemplated from the Analysis of Hierarchical Processes (AHP), which determine the level of hierarchy. Preferences are exponential weights, which gives the advantage of increasing or decreasing the perception or acceptance of quality indices, according to the established hierarchy criteria. It seeks to contribute to the construction of carbon bond valuation mechanisms that take into account the environmental problems associated with each country, its level of development, social inequality, needs and preferences.
- The advantages of the present work are the following:
- This model then allows incorporating the objective and subjective assessment of the attributes analysed. In this way, normative limits, preferences, needs and efficiency criteria can be incorporated according to the situation analyzed. It has no metric compatibility problems between the indexes, since it is not necessary to translate them through a subjective economic cost (a priori). This methodology allows the incorporation of PSO Particle Swarm Optimization (Camargo et al., 2018; Camargo, 2019, 2022b) and has the versatility to be applied to any problem.
- The associated Intrinsic (marginal) Cost (IC) is nothing more than the slope of the efficiency curve (investment cost and CO_2 emissions) of the fuzzy functions, which are obtained from the indices (Camargo et al., 2018). This gives us the (marginal) effect of worsening or improving one index over the other. This is useful for establishing penalty mechanisms for negative externalities or incentives for positive externalities. The determination of this cost is obtained by the method in a relatively simple way and, except for the determination of preferences, it is not subject to the subjectivity of the evaluator. T-norms that are differentiable are used.
- It allows to know the elasticities of the demand and, it is not necessary to know them in advance. Also, it does allow a ceteris paribus analysis since the functional relationships obtained are from possibilistic models (they contemplate uncertainties) and the relationships are not direct.

This work is organized as follows: in Section 2, the state of the art is presented. In Section 2. the material and methods are presented. The model of productive chain and Marginal evaluation model of the productive chain by means of intrinsic cost and AHP is developed in Section 2.1. The Intrinsic Cost Index for each t-norm are described in Section 2.2. The t-norm used in the fuzzy intersection (confluence) are: Einstein's Product (Section 2.2.1.), Algebraic Product (Section 2.2.2.), the particular (Section 2.2.3.) and generic form of Hamacher's Product (Section 2.2.4.). Lastly, in Section 3, a simulation of the Argentine productive chain with an evaluation scenario-based is presented: Model parameters, Analytic Hierarchy Process (AHP) and Exponential Weights (Section 3.1), the case of decreasing emissions while costs grow (Section 3.2), decrease in emissions while costs decrease (Section 3.3), analysis of the Intrinsic Cost index, effect of varying the efficiency (both perspectives) (Section 3.4) and the comparison of t-norms (Section 3.5). The conclusions are described in Section 4 and in the annex a comparison of the IC is presented.

2. Materials and methods

2.1. Model of Argentine Productive Chain (APC), Fuzzy Decision Making, Analytic Hierarchy Process (AHP) and Marginal evaluation model by means of intrinsic cost

The Argentine Productive Chain (APC) is considered (Figure 1), whose sectors of the production chain are the following: extraction, processing, manufacturing, construction, transport and waste treatments, the latter is distributed throughout the entire system. The transport sector is considered concentrated; this is like one more stage of the Life Cycle Analysis (LCA). This LCA is carried out for two inputs: materials used and fuel required in the following stages: resource extraction, material processing, manufacturing, construction, transportation and waste management (Camargo, 2019, 2021, 2022a, b). The parameters (and technical data) were mainly

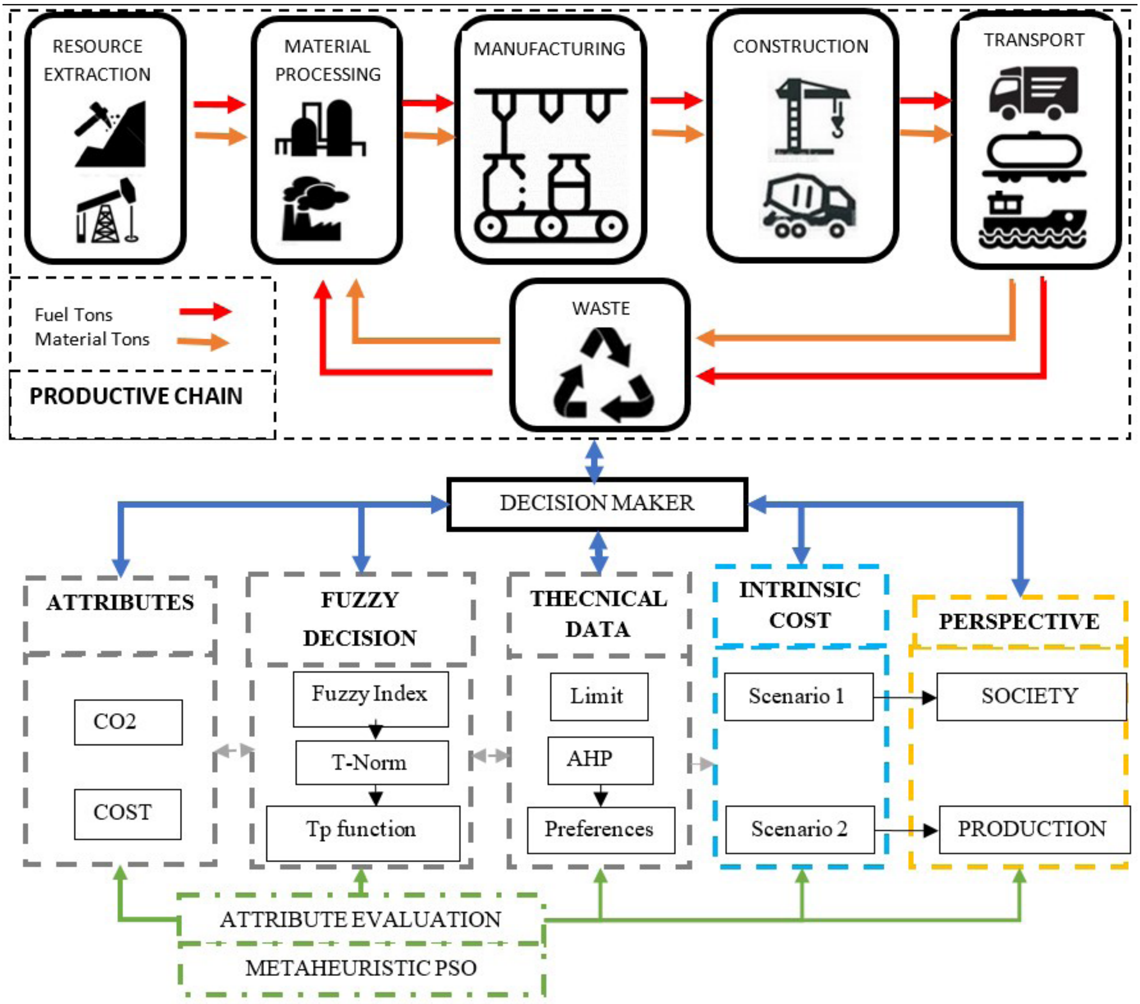

Figure 1. Life Cycle Analysis (LCA) model applied to the Argentine Productive Chain (APC), Analytic Hierarchy Process (AHP) and marginal evaluation model by means of intrinsic cost. Source: The author.

processed and the complete model was validated from the information of the public reports prepared by the public database available from the Ministry of Energy and Mining (Argentina, 2022).

The decision-maker then evaluates the resulting LCA energy balance indices (emissions and investment cost), and he transforms them to the fuzzy domain taking into account upper and lower limits obtained from extreme LCA analyses. This is by determining the efficiency index to be optimized (fuzzy intersection). Based on the production chain model (LCA model), the attributes to be evaluated (investment cost and CO_2 emissions) are established. Additionally, the preferences (AHP) and upper and lower limits (statics) of these attributes are established. From there, the evaluation of the indices is carried out, which also depend on the sense of improvement of the functional (growth or decrease).

Two scenarios for reduce the environmental impact (emissions) are analyzed. These scenarios are performed according to the perspective of the decision maker about the Emissions and Investment Cost (Carbon Price). The points of these scenarios were obtained for constant efficiency levels (constant t-norm) through the Particle Swarm Optimization (PSO). The first scenario is associated with the perspective of society, seeking to protect the most vulnerable sectors. In this sense, it seeks to increase the investment costs. This is achieved by implementing carbon capture and waste treatment methods. In the second scenario, the perspective of the productive chain (entrepreneur) is analyzed. In this case, the aim is to diminish the cost investment costs, seeking productive efficiency. This is achieved by seeking to increase the energy efficiency of the system, with the least possible equipment. Additionally, an aspect that influences the improvement of energy efficiency is the improvement of transport systems. The model was calibrated and validated using the energy records made by the Argentine state and the CO_2 emissions market records (Argentina, 2022; Investing.com, 2022; Camargo, 2022b).

Figure 2 presents the Analytic Hierarchy Process (AHP) method, which is used in this method. The process requires the decision maker to provide proportions subjective evaluations regarding the relative importance of each of the attribute. A hierarchical structure is constructed for the prequalification criteria and the contractors wishing to prequalify for the model (Liu et al., 2020; Amenta et al., 2021).

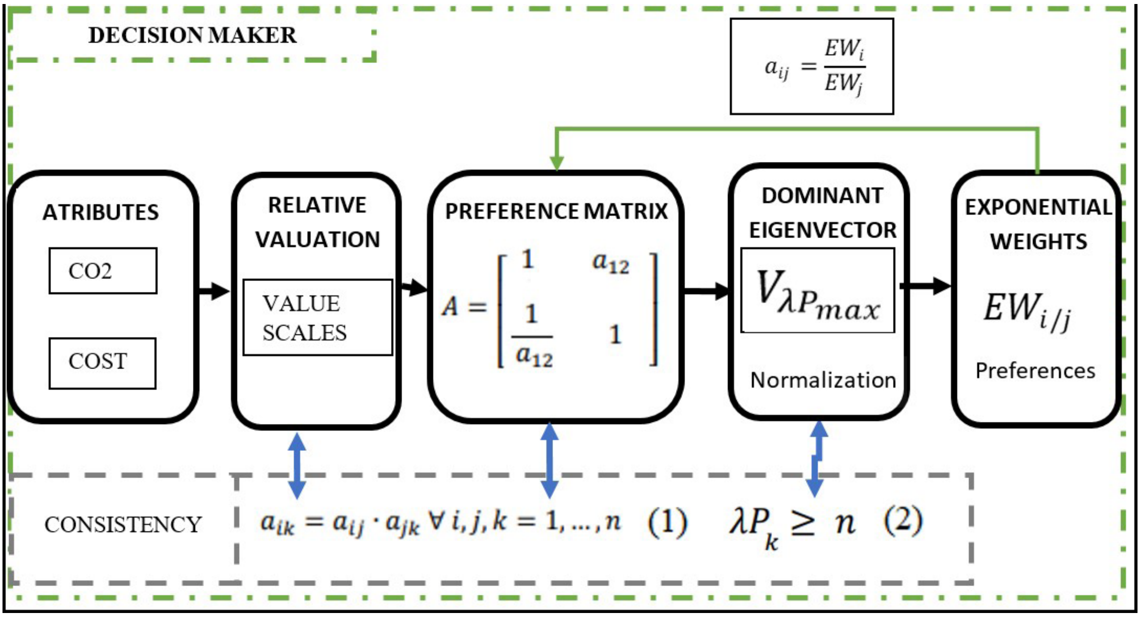

Figure 2. Analytic Hierarchy Process (AHP) applied to the Argentine productive chain. Source: The author.

The attributes (investment cost and carbon dioxide emissions) are valued relatively, according to a scale of values (between 1 and 9). This value implies that one attribute is so many times greater than the other, according to the scale of values that has been established. These values are stored in a matrix called preference matrix, where it is satisfied that: $a_{ij} = 1/a_{ji}$ or (consistency condition (1)). In this matrix, the first row and column correspond to the emissions and the second row and column correspond to the cost of emissions. Obviously, if the preference of an attribute is compared with itself, it will be unity, so the diagonal of the matrix will be unity.

From this matrix, the Perron eigenvalues and eigenvectors λP_k are obtained. There is perfect consistency if these eigenvalues λP_k are worth the order of the matrix (n). Otherwise, they must have at least that value. From these consistency criteria, other consistency indices are proposed (consistency condition (2)), which ensure compliance with the axioms in the weighting of attributes, according to decision theory (transitivity, completeness, asymmetry and difference symmetry). The alteration of this matrix causes it to lose coherence (consistency), with less compliance with the aforementioned axioms. The indexes that measure this consistency are called consistency indexes. This alteration can be made to make preferences more flexible (Exponential Weights or EW), in case a laxer decision maker is sought, at the cost of losing consistency in his decision making. For more details on this, the following references present more information on this (Saaty, 2003; Schweickardt & Miranda, 2009; Camargo, 2019; Liu et al., 2020).

Since the preference matrix satisfies the mathematical axioms of Perron's theorem (Saaty, 2003; Liu et al., 2020), it is spoken of Perron's eigenvectors. The dominant eigenvector (eigenvector whose eigenvalue is dominant or mostly positive) is taken, which are normalized, so as not to overvalue or underestimate the attributes. From this, the exponential weights of the preference functions are determined, which expand or contract the fuzzy indices.

From these weights, the preference matrix can be obtained again, dividing them $a_{ij} = \frac{EW_i}{EW_j}$ (see Equation (9)), where i 1 is the CO_2 emissions and $j = 2$ is the investment cost. For this reason, the preference matrix has perfect consistency (in addition to the fact that only two criteria are analyzed).

The Exponential Weights then model the decision maker's preferences regarding the evaluated attributes. The exponential weights then model the decision maker's preferences regarding the evaluated attributes. That is, depending on the assigned hierarchy, the indexes are over- or under-evaluated. This over-evaluation or under-evaluation will impact the economic evaluation (tax or carbon price).

Fuzzy Decision Making Theory is based on human behavior to make decisions (decision maker) based on a criterion of preferences (Exponential Weights) and valuation under uncertainty (fuzzy function). In this theory, there is a set of indices that are transformed to the fuzzy domain by means of an acceptance (fuzzy) function. The transformation algorithm to the fuzzy domain and confluence is presented, according to the FDM. In Equation 2, the $\mathrm{EW_m}$ are Exponential Weights (for each attribute m), whose effect is to expand ($\mathrm{EW_m} < 1$) and contract ($\mathrm{EW_m} > 1$) the fuzzy functions (Camargo et al., 2018; Camargo, 2022b). If the index is desired incremented, the function has a positive slope (and vice versa). The $\mathrm{EW_m}$ are obtained from the AHP (Schweickardt & Miranda, 2009; Schweickardt & Pistonesi, 2010; Camargo, 2019, 2022b). The attributes associated with Investment Cost and CO_2 will have a dilation in their fuzzy functions, while the others will have no effect.

BEGIN /* Fuzzy Decision Making */

Data: Objective and Constraints U_m, Exponential Weights EW_m (AHP), Lower U_m^{Low} and Upper U_m^{Up} Limits.

FOR $(m = 1:2)$ **DO**

Step 1: Calculate the auxiliary variable β.

$$\beta = \begin{cases} 1 & U_m \quad growth \\ 0 & decrease\,of\,U_m \end{cases} \tag{1}$$

Step 2: Calculate the states μ_m using the next function.

$$\mu_m = \begin{cases} \beta & ,U_m^{Low} \geq U_m \\ \left(\left(\dfrac{U_m^{Up} - U_m}{U_m^{Up} - U_m^{Low}} \right) \cdot (1-\beta) + \left(\dfrac{U_m - U_m^{Low}}{U_m^{Up} - U_m^{Low}} \right) \cdot \beta \right)^{EW_m} & ,U_m^{Low} \leq U_m \leq U_m^{Up} \\ 1-\beta & ,U_m^{Up} \leq U_m \end{cases} \tag{2}$$

END FOR

Step 3: Calculate $\mathrm{tp}\left(\mu_i, \mu_j\right)$ using the chosen t-norm, where $i = 1$ is the CO_2 emissions and $j = 2$ is the Investment Cost.

IF Algebraic Product **THEN**

$$\mathrm{tp}\left(\mu_i, \mu_j\right) = \mu_i \cdot \mu_j \tag{3}$$

ELSE IF Einstein's Product **THEN**

$$\mathrm{tp}\left(\mu_i, \mu_j\right) = \frac{\mu_i \cdot \mu_j}{2 - \left(\mu_i + \mu_j - \mu_i \cdot \mu_j\right)} \tag{4}$$

ELSE IF Particular Hamacher's Product **THEN**

$$\mathrm{tp}\left(\mu_i, \mu_j\right) = \frac{\mu_i \cdot \mu_j}{\mu_i + \mu_j - \mu_i \cdot \mu_j} \tag{5}$$

ELSE IF Generic Hamacher's Product **THEN**

$$\mathrm{tp}\left(\mu_i, \mu_j\right) = \frac{\mu_i \cdot \mu_j}{p + (1-p) \cdot \left(\mu_i + \mu_j - \mu_i \cdot \mu_j\right)} \tag{6}$$

END IF

END PROGRAM

To simplify the algorithm, an auxiliary variable β is added to establish the fuzzy functions (Equation 1 and Equation 2), depending on whether it is growth or decrease (Camargo, 2022b). The preference function index μ_m (Equation 2) is associated with the degree of acceptance of the evaluated index by the decision maker (economic cost and CO_2 emisions), according to his established preferences (Exponential Weights or EW). For each objective (or restriction), calculated in the previous section, the fuzzy functions associated to their objective or restriction are defined as follows: consider an upper and a lower limit in the possible values of the variable corresponding to a certain objective or constraint m, U_m (Shahzadi et al., 2020; Camargo et al., 2018; Camargo, 2022b).

The most common confluence (fuzzy operator), is the fuzzy intersection or t-norm product. Then, $tp\left(\mu_i, \mu_j\right)$ is an operator (called, in general, t-norm) between values of membership functions. In the state of the art there are multiple t-norms: algebraic product (Equation 3), Einstein's product (Equation 4) and Hamacher's product (Equation 5 and Equation 6). These t-norms fulfil the interesting property of being differentiable, which does not meet the t-min for example. This property allows us to obtain the impact of an objective analyzed in the others, that is, the social cost. The constant P (Equation 6) is a parameter that defines a family of curves depending on its value. As a consequence, the final intersection will be more demanding or lax depending on its value (Camargo et al., 2018; Camargo, 2022b; Shahzadi et al., 2020).

The present line of research obtains the economic valuation of each criteria through the Intrinsic Cost index (Equation 7 in Section 2.2.). This index is associated with the economic value of an index, which is not directly monetizable. A first version of this index which was presented in the next reference (Schweickardt & Pistonesi, 2010), and an improved version was presented in the next reference (Camargo et al., 2018) only been obtained in the past for the Einstein's Product t-norm and with another indices of the electrical power systems. This method derives the variable U_j, which is related with the criterion j (Investment Cost), with respect to the variable U_i, which is generally related to index i (Emissions). The intrinsic cost index (IC) takes into account the economic valuation of non-monetizable indices (Section 2.2.), which is based on an Objective valuation (incremental valuation based on extreme values), subjective and hierarchical (AHP). The IC corresponds to the derivative of one index with respect to the other (Cost of Emission with respect to Emissions), which is not easy to determine due to the presence of uncertainty and subjectivity.

The latter will influence the sign of the index, determining whether it is a penalty or an incentive. This last will be explained in section 3.1 and section 3.2. The intrinsic cost allows readjusting the value of the references, if necessary. This is by comparing the marginal cost obtained with carbon credit prices in the market.

2.2. Intrinsic Cost Index of CO emissions based on Fuzzy Decision Making Theory

The Intrinsic Cost (IC) index is in the Equation 7, where $i = 1$ is the CO_2 emissions and $j = 2$ is the Investment Cost.

$$IC_{ji} = \frac{\partial U_j}{\partial U_i} = \frac{\partial U_j}{\partial \mu_j} \bullet \frac{\partial \mu_j}{\partial \mu_i} \bullet \frac{\partial \mu_i}{\partial U_i} \tag{7}$$

Since it is the derivative, then it is associated with the slope of the curve $U_j = f\left(U_i\right)$. The terms $\frac{\partial \mu_{i/j}}{\partial U_{i/j}}$ (depending on whether i or j) are those derived of preference functions (Equation 8).

$$\frac{\partial \mu_{i/j}}{\partial U_{i/j}} = \pm \frac{EW_{i/j}}{U_{i/j}^{Up} - U_{i/j}^{Low}} \bullet \mu_i^{1 - \frac{1}{EW_{i/j}}} \tag{8}$$

IC_{ij} then represents the slope of the efficiency frontier of the functional relationship between the designated attributes. The sign $\pm$ depends on whether it's a growth (+) or decrease (-). The term $\frac{\partial \mu_j}{\partial \mu_j}$ corresponds to the derived of the fuzzy function $\partial \mu_j$ respect to the fuzzy function $\partial \mu_j$. If both attributes (i and j) correspond to growth or decrease then it will have a (+) sign, in any other case it will be (-).

The generic definition of IC is presented in the Equation 9.

$$IC_{ij} = \underset{sign}{\underset{\Downarrow}{\pm}} \; \underset{a_{ij}}{\underset{\Downarrow}{\left(\frac{EW_i}{EW_j}\right)}} \; \underset{b_{ij}}{\underset{\Downarrow}{\left(\frac{U_j^{Up} - U_j^{low}}{U_i^{Up} - U_i^{low}}\right)}} \; \underset{c_{ij}}{\underset{\Downarrow}{\left(\frac{\mu_i^{1-\frac{1}{EW_i}}}{\mu_j^{1-\frac{1}{EW_j}}}\right)}} \; \underset{d_{ij}}{\underset{\Downarrow}{\left(\frac{\partial \mu_j}{\partial \mu_i}\right)}} \tag{9}$$

The term a_{ij} is the preference ratio associated with the Analytical Hierarchy Process (AHP), which gives the relative valuation between the evaluated attributes (CO_2 emissions and Investment Cost). Each value of a_{ij} is associated with the preference matrix of the attributes studied. The weights are obtained from an analysis of Perron's eigenvalue.

The term b_{ij} is the incremental cost of the an attribute j with respect to another of interest i, that is, it is associated with the absolute value of the attribute. The term c_{ij} is associated with the influence of the acceptance of attributes and preferences of decision makers. The term d_{ij} corresponds to the derived of the fuzzy function $\partial\mu_j$ respect to the fuzzy function $\partial\mu_j$ which is associated with the slope of the functional relationship of the preference functions. Factor b_{ij} determines the objective valuation (incremental cost) of attribute j, while a_{ij}, c_{ij} and d_{ij} determine the subjective valuation.

If it is true the Equation 10, then IC_{ij} will tend to infinity (its value can be very large).

$$EW_i \gg EW_j \; or \left(U_j^{Up} - U_j^{low}\right) \gg \left(U_i^{Up} - U_i^{low}\right) or \;\; \mu_j \gg \mu_i \tag{10}$$

This implies a very high valuation of attribute i respect to j. Therefore, the preferences and the limits should not have much difference. If it is possible, the preferences should be similar.

Additionally, if $EW_i = EW_j$ and $\mu_i = \mu_j$, then the Equation 11 is true.

$$IC_{ij} = \pm \frac{U_j^{Up} - U_j^{low}}{U_i^{Up} - U_i^{low}} = \pm b_{ij} \tag{11}$$

A constant value will be obtained, independent of the evaluated indices. In other words, the intrinsic cost will be an incremental cost between the extreme values. If additionally, the lower limits are null, then the Equation 12 is obtained.

$$IC_{ij} = \pm \frac{U_j^{Up}}{U_i^{Up}} \tag{12}$$

In other words, the IC will be equal to the average cost (or marginal cost) of the lower values. Therefore, based on this theoretical analysis, the proposed index can be applied as an economic indicator, to evaluate non-monetizable attributes and as a regulatory mechanism. Associated with the Coase theorem and property rights, it is the price at which the bonds must be traded according to the externality produced. Since it is a negative externality, it is a tax that the production chain must to pay.

2.2.1. Einstein's product t-norm

Since the Ceteris Paribus clause applies, then it is only necessary to compare two attributes, since all the others will be constant. Then, this work applies logarithm properties (Equation 13) to the definition of the confluence Einstein's product t-norm (Equation 4), and its mathematical derivative, the Equation 14 is obtained.

$$\log\left(tp\left(\mu_i, \mu_j\right)\right) = \log\left(\mu_i\right) + \log\left(\mu_j\right) - \log\left(2 - \mu_i - \mu_j + \mu_i . \mu_j\right) \tag{13}$$

$$0 = \frac{1}{\mu_i} + \frac{1}{\mu_j} - \frac{1}{2 - \mu_i - \mu_j + \mu_i . \mu_j} \bullet \left(-1 - \frac{\partial \mu_j}{\partial \mu_i} + \mu_j + \mu_i \bullet \frac{\partial \mu_j}{\partial \mu_i}\right) \tag{14}$$

Then, this work defines $\frac{\partial \mu_j}{\partial \mu_i}$ in the Equation 15.

$$\frac{\partial \mu_j}{\partial \mu_i} = -\frac{\mu_j}{\mu_i} \bullet \frac{\mu_j - 2}{\mu_i - 2} \tag{15}$$

The Intrinsic Cost of the Einstein Product and fuzzy ramp functions is presented in the Equation 16.

$$IC_{ij} = \pm \frac{EW_i}{EW_j} \bullet \frac{U_j^{Up} - U_j^{low}}{U_i^{Up} - U_i^{low}} \bullet \frac{\mu_i^{1 - \frac{1}{EW_i}}}{\mu_j^{1 - \frac{1}{EW_j}}} \bullet \frac{\mu_j}{\mu_i} \bullet \frac{\mu_j - 2}{\mu_i - 2} \tag{16}$$

The absolute value of the simplified Intrinsic Cost is presented in Equation 17.

$$sdx \tag{17}$$

2.2.2. Algebraic product

The next step is to derive the Algebraic product (Equation 3) respect to μ_i, it is applyiing the clause 'Ceteris Paribus', the Equation 18 is obtained.

$$\frac{\partial tp\left(\mu_i,\mu_j\right)}{\partial \mu_i}=\mu_j+\mu_i \bullet \frac{\partial \mu_j}{\partial \mu_i}=0 \tag{18}$$

Then, this work defines $\frac{\partial \mu_j}{\partial \mu_i}$ in the Equation 19.

$$\frac{\partial \mu_j}{\partial \mu_i}=-\frac{\mu_j}{\mu_i} \tag{19}$$

The Intrinsic Cost for Algebraic Product and fuzzy ramp functions is presented in the Equation 20.

$$IC_{i\,j}=\pm\frac{EW_i}{EW_j}\bullet\frac{U_j^{Up}-U_j^{low}}{U_i^{Up}-U_i^{low}}\bullet\frac{\mu_i^{1-\frac{1}{EW_i}}}{\mu_j^{1-\frac{1}{EW_j}}}\bullet\frac{\mu_j}{\mu_i} \tag{20}$$

The absolute value of the simplified Intrinsic Cost is presented in Equation 21.

$$IC_{i\,j}=\frac{EW_i}{EW_j}\bullet\frac{U_j^{Up}-U_j^{low}}{U_i^{Up}-U_i^{low}}\bullet\frac{\mu_j^{\frac{1}{EW_j}}}{\mu_i^{\frac{1}{EW_i}}} \tag{21}$$

2.2.3. Particular Hamacher's Product (PHP)

This work applies logarithm properties to the definition of the confluence Particular Hamacher's Product (Equation 5). The next step is to derive the Equation 22 respect to μ_i, it is applyiing the clause 'Ceteris Paribus', the Equation 23 is obtained.

$$\log\left(tp\left(\mu_i,\mu_j\right)\right)=log\left(\mu_i\right)+log\left(\mu_j\right)-log\left(\mu_i+\mu_j-\mu_i \bullet \mu_j\right) \tag{22}$$

$$0=\frac{1}{\mu_i}+\frac{1}{\mu_j}\bullet\frac{\partial \mu_j}{\partial \mu_i}-\frac{1}{\mu_i+\mu_j-\mu_i \bullet \mu_j}\bullet\left(1-\mu_j+\left(1-\mu_i\right)\bullet\frac{\partial \mu_j}{\partial \mu_i}\right) \tag{23}$$

Then, this work defines $\frac{\partial \mu_j}{\partial \mu_i}$ in the Equation 24.

$$\frac{\partial \mu_j}{\partial \mu_i}=-\left(\frac{\mu_j}{\mu_i}\right)^2 \tag{24}$$

The Intrinsic Cost for fuzzy ramp functions and Hamacher's Product (with $p=0$) t-norm is presented in the Equation 25.

$$IC_{i\,j}=\pm\frac{EW_i}{EW_j}\bullet\frac{U_j^{Up}-U_j^{low}}{U_i^{Up}-U_i^{low}}\bullet\frac{\mu_i^{1-\frac{1}{EW_i}}}{\mu_j^{1-\frac{1}{EW_j}}}\bullet\left(\frac{\mu_j}{\mu_i}\right)^2 \tag{25}$$

The absolute value of the simplified Intrinsic Cost is presented in Equation 26.

$$IC_{i\,j}=\frac{EW_i}{EW_j}\bullet\frac{U_j^{Up}-U_j^{low}}{U_i^{Up}-U_i^{low}}\bullet\frac{\mu_j^{1+\frac{1}{EW_j}}}{\mu_i^{1+\frac{1}{EW_i}}} \tag{26}$$

2.2.4. Generic Hamacher's Product (GHP)

Then, this work applies logarithm properties to the definition of the confluence Hamacher's (generic form) t-norm product (Equation 6). Then, the next step is to derive the Equation 27 respect to i, it is applying the clause 'Ceteris Paribus', the Equation 28 is obtained.

$$\log\left(\mathrm{tp}\left(\mu_i,\mu_j\right)\right)=\log\left(\mu_i\right)+\log\left(\mu_j\right)-\log\left(p+(1-p)\cdot\left(\mu_i+\mu_j-\mu_i\cdot\mu_j\right)\right) \tag{27}$$

$$\frac{1}{\mu_i}+\frac{1}{\mu_j}\cdot\frac{\partial\mu_j}{\partial\mu_i}-\frac{(1-p)}{p+(1-p)\cdot\left(\mu_i+\mu_j-\mu_i\cdot\mu_j\right)}\cdot\left(1-\mu_j+\left(1-\mu_i\right)\cdot\frac{\partial\mu_j}{\partial\mu_i}\right)=0 \tag{28}$$

Then, this work defines $\frac{\partial\mu_j}{\partial\mu_i}$ in the Equation 29 and Equation 30.

$$\frac{\partial\mu_j}{\partial\mu_i}=-\left(\frac{p+(1-p)\cdot\mu_j}{p+(1-p)\cdot\mu_i}\right)\cdot\frac{\mu_j}{\mu_i} \tag{29}$$

It is interesting to note the Equation 29 and Equation 30. In the first case ($p=1$), it is the intrinsic cost of the Algebraic Product (see Equation 19). In the second case ($p=0$), it is the intrinsic cost of the Particular Hamacher's Product (see Equation 24). This is logical, since if these values of p are replaced in Equation 6, the respective t-norms are obtained. Then the intrinsic cost associated with the PHP t-norm behaves as an intermediate case to these two t-norms (AP and GHP).

$$\frac{\partial\mu_j}{\partial\mu_i}=\begin{cases}-\frac{\mu_j}{\mu_i}, & p=1\\ -\left(\frac{\mu_j}{\mu_i}\right)^2, & p=0\\ -\left(\frac{p+(1-p)\cdot\mu_j}{p+(1-p)\cdot\mu_i}\right)\cdot\frac{\mu_j}{\mu_i}, & \text{another case}\end{cases} \tag{30}$$

The Intrinsic Cost for fuzzy ramp functions and Hamacher's product (generic form) t-norm is presented in the Equation 31.

$$IC_{ij}=\pm\frac{EW_i}{EW_j}\cdot\frac{U_j^{Up}-U_j^{low}}{U_i^{Up}-U_i^{low}}\cdot\frac{\mu_i^{1-\frac{1}{EW}}}{\mu_j^{1-\frac{1}{EW}}}\cdot\left(\frac{p+(1-p)\cdot\mu_j}{p+(1-p)\cdot\mu_i}\right)\cdot\frac{\mu_j}{\mu_i} \tag{31}$$

The absolute value of the simplified Intrinsic Cost is in Equation 32.

$$IC_{ij}=\frac{EW_i}{EW_j}\cdot\frac{U_j^{Up}-U_j^{low}}{U_i^{Up}-U_i^{low}}\cdot\frac{\mu_j^{\frac{1}{EW_j}}}{\mu_i^{\frac{1}{EW_i}}}\cdot\left(\frac{p+(1-p)\cdot\mu_j}{p+(1-p)\cdot\mu_i}\right) \tag{32}$$

3. Simulation of the Argentine Energy System

According to section 2, two scenarios of environmental impact (emissions) reduction are analyzed. These scenarios are made according to the decision maker's perspective on Emissions and Investment Cost (Carbon Price). The points of these scenarios were obtained for constant efficiency levels (constant t-norm) through Particle Swarm Optimization (PSO) method.

The first scenario is associated with the perspective of society, seeking to protect the most vulnerable sectors. In this sense, it seeks to increase investment costs. This is achieved through the application of carbon capture and waste treatment methods. The second scenario analyses the perspective of the production chain (entrepreneur). In this case, the objective is to reduce investment costs by seeking productive efficiency. This is achieved by seeking to increase the energy efficiency of the system, with the least possible equipment. In addition, an aspect that influences the improvement of energy efficiency is the improvement of transportation systems. The model was calibrated and validated using energy records made by the Argentine State and CO_2 emissions market records (Argentina, 2022; Investing.com, 2022; Camargo, 2022b).

The curves are obtained from the solutions obtained with the PSO for different efficiencies and intersection operators (t-norms). In section 3.2. the societal perspective is studied, while in section 3.3. the production chain perspective is studied, in section 3.4. a comparison is made of the intrinsic costs in both scenarios and finally in section 3.5. an overall comparison is made of the curves for the different t-norms, with respect to the indices (emissions and investment costs) and the intrinsic cost (IC).

The analyses are first presented for each t-norm separately (Section 3.2 and Section 3.4), for different reasons. First, to validate the curves calculated by the OSP and the developments made for each intrinsic cost and to show that they are logical. It is necessary to verify that the intrinsic cost corresponds to the slope of the investment cost vs. emission efficiency curves. It is also a matter of verifying the carbon prices obtained and comparing them with those established internationally. Secondly, to facilitate the comparison in section 3.4. Finally, it seeks to set a precedent for future work, incorporating state-of-the-art information that can be used for future improvements, proposals and/or comparisons. All of this is in line with the stated objective of the article: to compare the novel economic valuation models to establish a price for emissions that will allow us to evaluate their externality and provide tools for environmental regulation.

3.1. Model parameters, Analytic Hierarchy Process (AHP) and Exponential Weights

The main parameters of the model are presented in this section, the application of the Analytic Hierarchy Process (AHP) procedure and Exponential Weights. For emissions, the upper limit is $U_i^{Low} = 0.3 \frac{\text{t CO}_2\text{ eq}}{\text{MWh}}$ and the lower limit is $U_i^{Up} = 0.4 \frac{\text{t CO eq}}{\text{MWh}}$ (both are per year). For costs, the upper limit is $U_j^{Low} = 0.5 \frac{\text{USD}}{\text{MWh}}$ and the lower limit is $U_i^{Up} = 2.5 \frac{\text{USD}}{\text{MWh}}$ (both are per year). Where $i = 1$ is the CO_2 emissions and $j = 2$ is the Investment Cost.

A parameter $P = 0.5$ was chosen to represent a perfectly intermediate case between the AP and the particular HP. Other smaller (closer to AP) and larger (closer to (particular HP) values can be estimated.

Preferences (exponential weights) are proposed that seek a middle ground between the perspective of the production chain and society. To determine the exponential weights (Equation 38), the procedure shown in Figure 2 is carried out: the preference matrix (Equation 33), its eigenvalue (Equation 34) and eigenvector (equation 35), the dominant eigenvalue (Equation 36), the dominant eigenvector (Equation 37) and the exponential weights (Equation 38). The exponential weights are obtained from the normalization (from the highest value of the eigenvector) of the eigenvector associated with the dominant eigenvalue.

The matrix A is positive definite and reciprocal. Moreover, this matrix has not been relaxed in this work, so the consistency is perfect. Therefore, the consistency indices are not presented in this paper.

$$A = \begin{bmatrix} 1 & 2 \\ 0.5 & 1 \end{bmatrix} \tag{33}$$

$$\lambda p = \begin{bmatrix} 2 0 \end{bmatrix} \quad \Rightarrow eigenvalue(A) \tag{34}$$

$$V = \begin{bmatrix} 0.89 & -0.89 \\ 0.44 & 0.44 \end{bmatrix} \Rightarrow eigenvector(A) \tag{35}$$

$$\lambda p_{max} = 2 \quad \Rightarrow dominant\, eigenvalue(A) \tag{36}$$

$$V_{\lambda p_{max}} = \begin{bmatrix} 0.89 \\ 0.44 \end{bmatrix} \Rightarrow dominant\, eigenvector(A) \tag{37}$$

$$EW = normalize\left(V_{\lambda p_{max}}\right) = \begin{bmatrix} 1 \\ 0.5 \end{bmatrix} \Rightarrow Exponential\, Weights \tag{38}$$

It is observed that the exponential weight (and the component of the eigenvector) associated with CO_2 emissions is twice (a_{12}) the weight associated with cost (and vice versa). This translates into the preference matrix, which can be obtained and interpreted from this relationship. It is worth mentioning that if matrix A is transposed and the same procedure is performed on this transposed matrix, similar results will be obtained, but with inverse priority. That is, investment will have twice the importance of emissions. This comparison can be made in future work.

All these parameters were calibrated from simulations of extreme cases, previous works and contributions (Argentina, 2022; Camargo, 2022a, b).

3.2. Scenario 1: decrease in emissions while investment cost grows (society perspective)

First, we analyse the case of a decrease in emissions while the cost of investment in the Argentine production chain increases. This is a case of positive externality from the production chain to the users, since the investments made favor society with a reduction in emissions. The effect of quality variation is presented in Figures 3 and Figure 4. This represents the variation of the proposed requirement limits, which means that if an index worsens, it is more difficult to compensate it. The costs per MWh obtained are in accordance with those established by the Argentine Renewable Energy Chamber and the Ministry of Energy (Argentina, 2022): renewable projects cost around $1.5\frac{USD}{MWh}$.

In the first of these graphs, it is observed that as the required index resulting from the confluence increases, it is more difficult to meet this requirement, to the point that this requirement is met only with one investment value. Therefore, as the efficiency level of the proposed solution (t-norm) increases, it is more difficult to meet and, therefore, the higher the investment cost required to achieve the same level of efficiency. If the value of the confluence (efficiency) increases, higher investments are required to obtain the same resulting emission. This

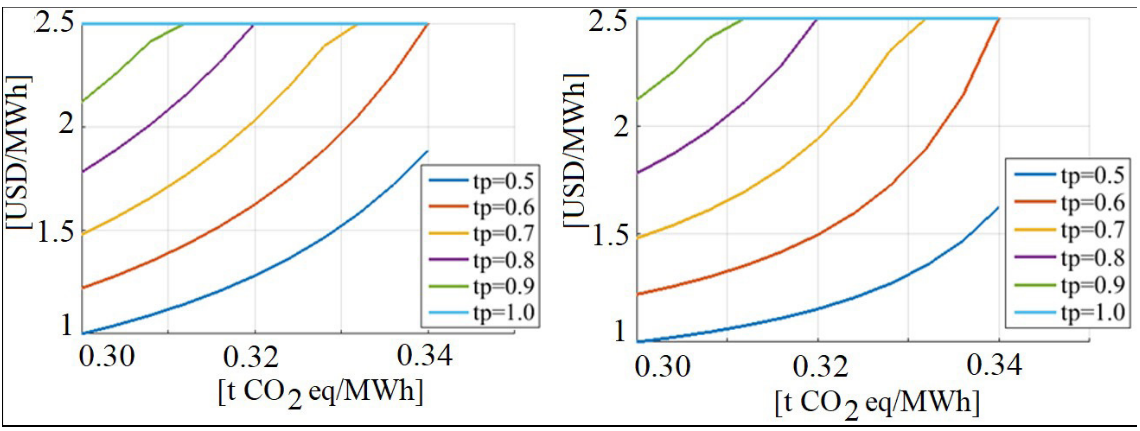

Figure 3. Investment Cost vs. Emissions for the Algebraic Product and Particular Hamacher Product. Source: The author.

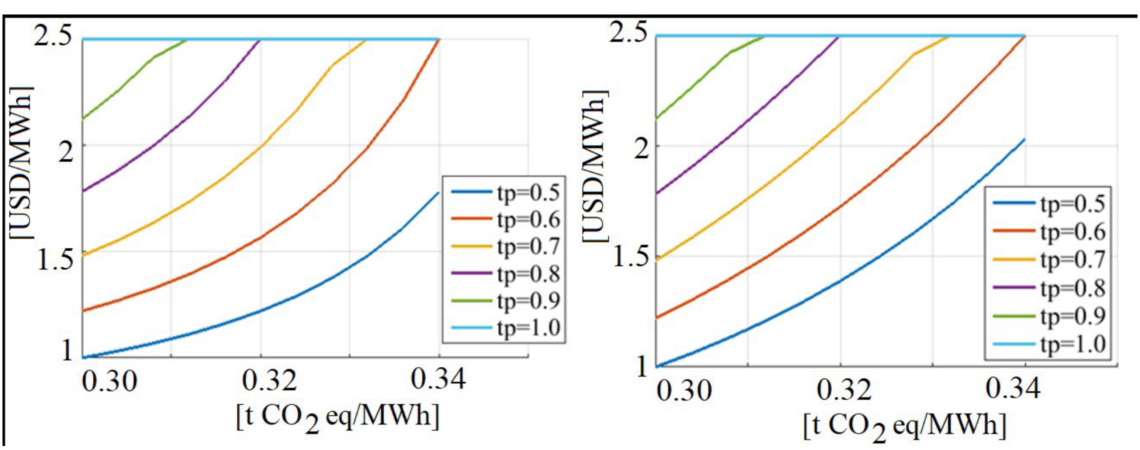

Figure 4. Investment Cost vs. Emissions for the General Hamacher's Product ($p = 0.5$) and Einstein's Product. Source: The author.

occurs up to a point where there will no longer be alternatives to achieve a better solution, since a maximum efficiency level (unity) has been reached. The limit established for the investment cost (given this minimum efficiency) is $2.5\frac{USD}{MWh}$.

3.3. Scenario 2: decrease in emissions while investment cost decrease (productive chain perspective)

The case of the reduction of emissions while decreasing investment costs in the Argentine production chain is analyzed. This is a case of negative externality, since the investments made favor society with the decrease in emissions. The Intrinsic Cost (IC) will be negative (this is according to what has been developed mathematically), however, the absolute value is taken. The perspective with respect to the production chain, i.e., it is analysed to reduce emissions with the lowest possible investment. The effect of the resulting confluence variation is presented in Figures 5 and Figure 6.

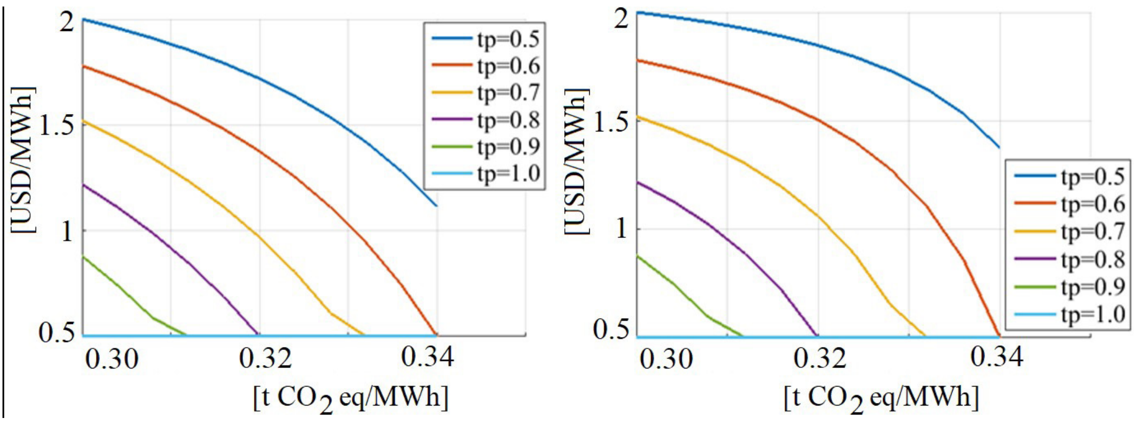

Figure 5. Investment Cost vs. Emissions for the Particular Hamacher's Product (PHP). Source: The author.

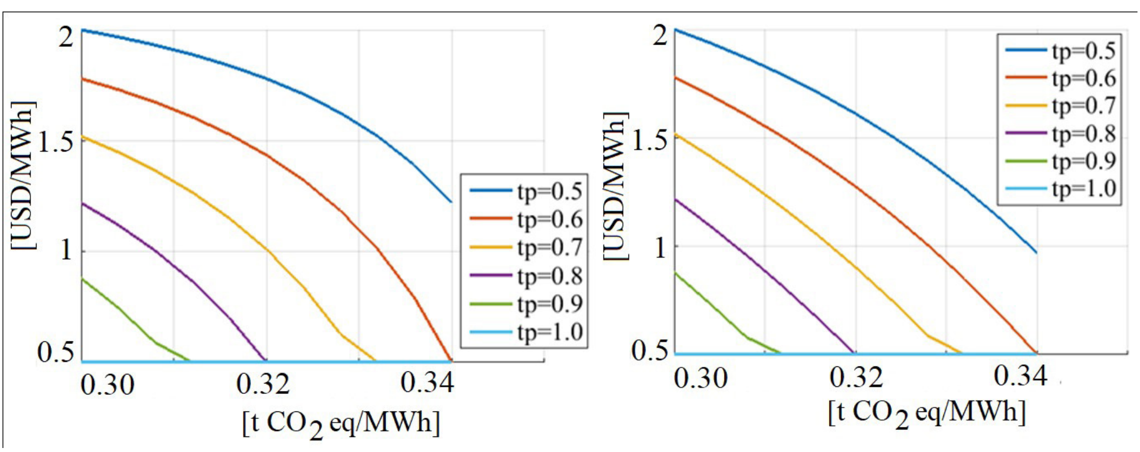

Figure 6. Investment Cost vs. Emissions for the Einstein's Product. Source: The author.

The first of these graphs shows that as the confluence value increases (*tp* value), less investment is required to obtain the same resulting emission, it is for the Particular Hamacher's Product (PHP). That is, by improving the quality of the proposed solution (fuzzy intersection *tp*), the cost of addressing such resolution is reduced, which is logical. Therefore, the PSO metaheuristic acts logically with respect to the developed model (emissions vs. investment cost). Section 3.5. discusses which is the most demanding t-norm when comparing them in the same figure, which makes the analysis more effective.

It is observed that the curves have analogous conclusions, varying their slope and therefore their rigor. The results are logical and in principle, the curves calculated by the PSO are correct.

3.4. Comparison of Intrinsic Cost: effect of varying the efficiency (both perspectives)

The Intrinsic Cost (IC) will be positive (this is according to what has been developed mathematically), however, the absolute value is taken. The absolute value is taken to simplify the analysis, since both indices have the same magnitude, but different signs. However, the sign is interpreted in the analysis as the impact that the increase or decrease in emissions will have according to the analyzed perspective. Any increase in emissions will be a negative externality (society perspective) and any reduction of emissions will be a positive externality (supply chain perspective). The slope of the IC curve is positive and, therefore, any increase in emissions will be penalized (society perspective) or incentivized (supply chain perspective).

In Figure 7 and Figure 8 the final IC values are between $20\frac{USD}{t\ CO2\ eq}$ and $66\frac{USD}{t\ CO2\ eq}$ (AP), $18\frac{USD}{t\ CO2\ eq}$ and $110\frac{USD}{t\ CO2\ eq}$ (PHP), $19\frac{USD}{t\ CO2\ eq}$ and $85\frac{USD}{t\ CO2\ eq}$ (GHP) and $22\frac{USD}{t\ CO2\ eq}$ and $47\frac{USD}{t\ CO2\ eq}$ (EP). Then, the global values of IC are a minimum of $18\frac{USD}{t\ CO2\ eq}$ and a maximum of $110\frac{USD}{t\ CO2\ eq}$. Since 2020, the historical values of carbon bond prices are a minimum of $20\frac{USD}{t\ CO2\ eq}$ and a maximum of $100\frac{USD}{t\ CO2\ eq}$, so the prices obtained are in line with international values (Investing.com, 2022). With this price, carbon bonds are traded, which allow investment in various green energy projects.

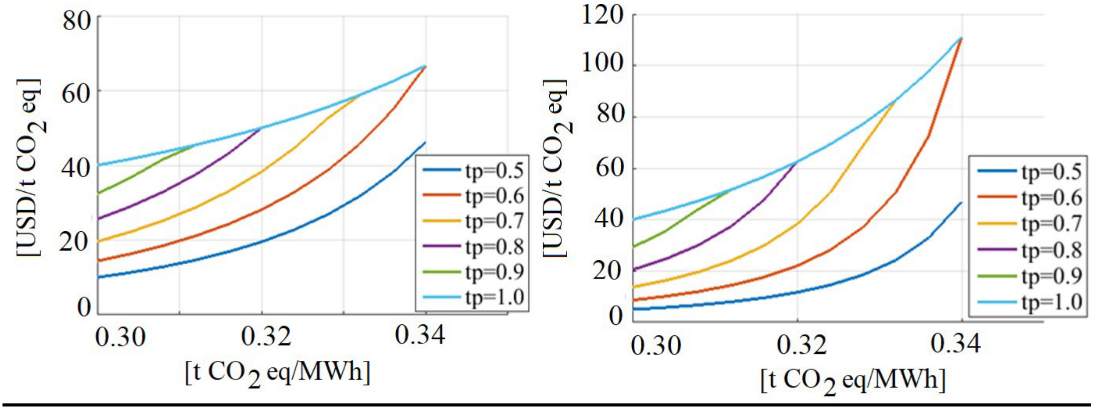

Figure 7. Investment Cost vs. Emissions for the Algebraic Product and Particular Hamacher's Product. Source: The author.

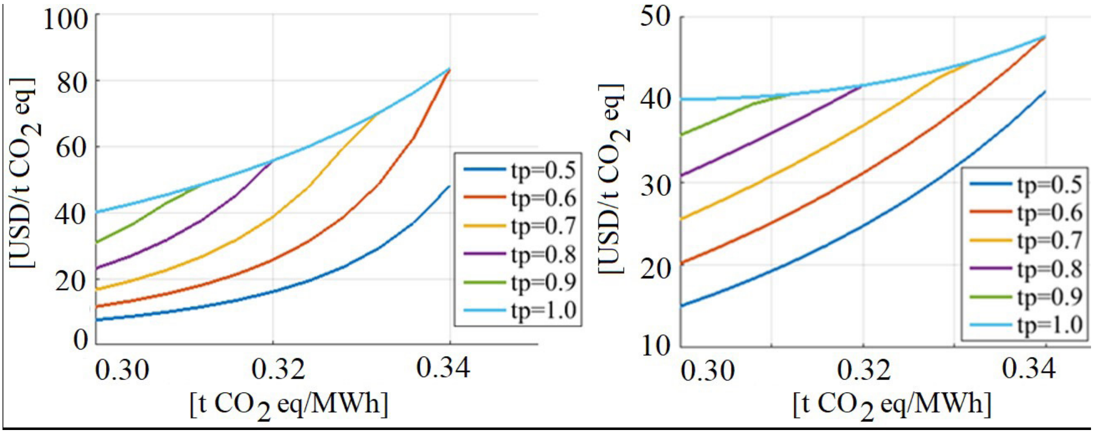

Figure 8. Investment Cost vs. Emissions for the General Hamacher's Product and Einstein's Product. Source: The author.

Taking into account the first of these graphs, for example, it corresponds to both scenarios (in absolute value). The impact on the economic valuation by varying the efficiency indexes (tp) is analyzed, given the solutions obtained by the PSO. As the efficiency level increases, the IC will be higher until it corresponds to the only possible solution (unit efficiency in tp). Additionally, a higher economic valuation will have this solution and therefore a greater economic incentive on the part of the regulator. Section 3.5. discusses which is the most demanding t-norm when comparing them in the same figure, which makes it more effective to analyse.

The curves have a maximum efficiency cap, which is given by the established tp value (light blue curve of $tp = 1$), and therefore none can mathematically exceed said curve. Additionally, it is observed that (at least in absolute value), the functions are increasing. This is related to exponential weights and can be further developed in future works. Efficiency values below 0.5 are very difficult to meet (in the analysis presented in this work), especially in the intrinsic cost, for that reason lower efficiency values were not studied.

Therefore, the results are shown to be consistent and comparison between the different curves can be made in the Section 3.5.

3.5. Comparison of t-norms

Once the coherence and logical basis of the curves obtained from the mathematical developments of the indices developed has been demonstrated, the pertinent comparisons are made, gathering as much information as possible in different graphs. Figure 9 shows the comparison between the graphs of the t-norms with the investment cost and emissions indexes evaluated (left graph corresponds to the investment cost (increase) vs. CO_2 (decrease) and right graph corresponds to the Investment Cost (decrease) vs. CO_2 (decrease)). A comparison of the resulting curves with constant confluence is presented, for different types of t-norm. It is observed that with respect to the curve of the analyzed attributes, the most demanding t-norm is the Einstein's product and the least demanding is the Hamacher's Product. These two operators are the most complicated to calculate, given the number of operations to be performed. A value of $tp\left(\mu_i, \mu_j\right) = 0.6$ is taken (Camargo et al., 2018; Camargo, 2022b), which gives the most interesting results (average efficiency) value which is obtained by the PSO in this methodology).

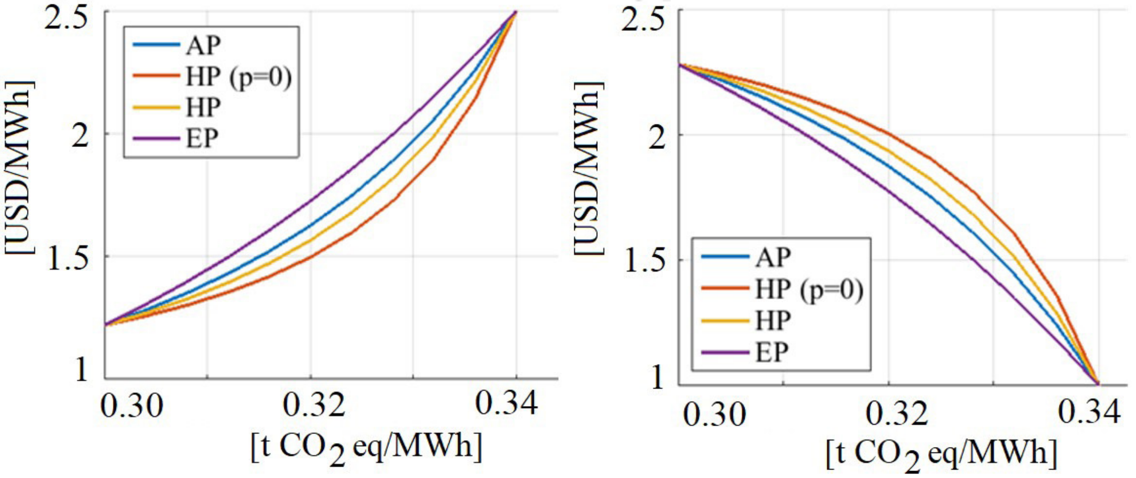

Figure 9. Left graph: Investment Cost (increase) vs. CO_2 (decrease). Right graph: Investment Cost (decrease) vs. CO_2 (decrease). Source: The author.

As for the left graph, the most demanding t-norm is the Einstein product, since it requires a higher investment cost for the same quality index. As for the second graph, it is observed that for high values of emissions, the highest ICs will be for the t-norm Hamacher's particular product, while the least demanding one is the t-norm Einstein's product. In the left graph, it is observed that the slopes of the functions are positive, while in the right graph they are negative. This is due to the concept of dominance; the best solutions are those that increase the investment cost (left graph) or decrease it (right graph). Solutions that are above the curve (Left graph) or below (Right graph) are dominated solutions.

Regarding the IC, it is observed that up to a certain value (or range) of CO_2 the Einstein Product is the most demanding and the Hamacher Product (p=0) is the least demanding. For higher values of CO_2 emissions, the mentioned hierarchy is reversed. This level of demand is observed up to a cost of almost 40 USD/MWh. Then the hierarchy is inverted, however, in this case the aim is to reduce emissions and therefore the previous hierarchy may be useful.

This is associated with the value of the intrinsic cost of the Figure 10, which will be negative in the first case (negative slope) and positive in the second. However, to facilitate comparison, the absolute value has been obtained and this sign has been omitted. This sign will be important when establishing the incentive and penalty mechanism. Thus, in (Left graph) it is associated with the incentive, while in (Right graph) it is associated with the penalty (tax). If the IC is associated with a tax price (regardless of the sign), the production chain will be incentivized to reduce the resulting amount of emissions. That is: increase the investment made or improve energy efficiency. Thus, the IC is a good indicator of the price of a carbon market regulation mechanism and of the search for energy efficiency.

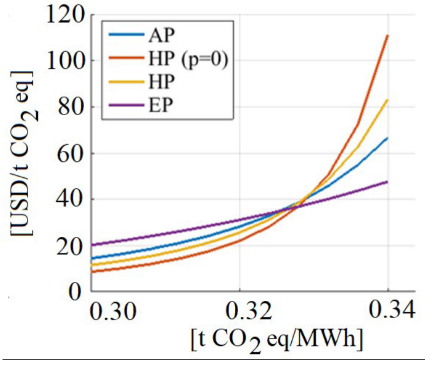

Figure 10. Intrinsic Cost vs. CO_2 Emissions (both cases: Figure 9). Source: The author.

Therefore, depending on the desired stringency in the regulation mechanisms, one t-norm can be chosen or another. It is observed that Einstein's product has the most uniform intrinsic cost, which can be an advantage to avoid excessive prices and high penalties. The t-norm that has the least variation in the IC is the Einstein Product, which makes it appropriate if the penalty or incentive is to be relatively constant or have little variation and not be progressive. The point of change in the requirement between the t-norm corresponds to the point of equality between the preference functions, in which case the intrinsic costs will be equal (see Annex 1).

The intrinsic cost at which all t-norms are equalized, at equal fuzzy indices in $i=1$ (Emissions) and $j=2$ (Investment Cost), is almost $40\frac{USD}{t\ CO2\ eq}$. Up to this point, the most demanding t-norm with respect to penalty cost is the EP (or the one that pays the best if it is a subsidy), while the least demanding with respect to penalty cost is the HP (or the one that pays the least if it is a subsidy). Similarly, after this point, the most demanding t-norm with respect to penalty cost is the HP with $P=0$ (or the one that pays the best if it is a subsidy), while the least demanding with respect to penalty cost is the PE (or the one that pays the least if it is a subsidy). Then the "best alternative" to use is the Einstein's product. However, this operator is the most difficult to compute (Equation 4).

This result has repercussions on the use that the decision-maker wants to make of the intrinsic cost to value non-monetizable attributes such as carbon dioxide emissions and thus establish regulation mechanisms. From society's point of view, if high emissions are desired, it is advantageous to use EP to have minimum penalties and taxes. Also, if low emissions are desired, it is advantageous to use EP to have low penalties in those cases, although they will be the highest in the case of high emissions. Therefore, if high stringency or incentives are required in the index regulation mechanisms, then the "best alternative" to use is the EP. However, this fuzzy operator and the IC are the most difficult to calculate.

So, there is no optimal t-norm, but there is "the most satisfactory one", which depends on the intended application of the proposed index. This shows a great versatility and practical application of the index to obtain the carbon price.

4. Conclusions

This work compared the economic valuation (Intrinsic Cost or IC) of carbon dioxide emissions (carbon price) of the Argentine production chain, which was associated with externality taxes (Kyoto Protocol). Two scenarios were studied according to the perspective of society or users and the perspective of the Argentine production chain (businessmen), which were calculated (all points of the corresponding graphs) using Particle Swarm Optimization (PSO). The experimental IC index and the applied methodology allowed incorporating objective (incremental) and subjective (acceptance, hierarchy and uncertainty) evaluation. In addition, this methodology allowed incorporating nonlinearity (no need for simplifications without metric compatibility problems).

The IC was applied to the attribute graphs, which are obtained from the combination of the Fuzzy Decision Making Theory (FDT), the Analytic Hierarchy Process (AHP) and the PSO with the following fuzzy operators: Algebraic Product (AP), Generic Hamacher (GHP), Particular Hamacher (PHP) and Einstein (EP). The absolute value is taken to conceptually analyse its value, without altering the graph (shortening the analysis since the graphs are the same in both scenarios except the sign), the sign is analyzed from the slope analysis. The sign indicates whether it is a subsidy or a tax.

The main results of this combination and its analysis are the follows.

First, the coherency of the obtained IC was demonstrated by mathematical and graphical analysis.

- From a mathematical point of view, the following points have been checked. Firstly, the variants of the IC index consider the relative valuation of preference weights (exponential) and ranking according to the AHP, the objective valuation of the non-monetizable attribute (differential cost) and its subjective valuation (preferences of the decision-maker). In addition, it has been found that the IC index depends on a non-monetizable attribute, its associated preferences, the type of membership function and the t-norm applied. There is an "inflection point" given by the equality in the fuzzy indices (see Annex 1) given by the equality in the fuzzy indices. This point defines the level of stringency of the t-norm, as it will be more or less difficult, depending on the desired results (investment cost and emissions). Moreover, the GPH t-norm is an intermediate case between the PA and the PPH, which was confirmed by obtaining the general expression of the IC for the GPH. Second, the AHP method successfully (and with perfect consistency) provided the requested (exponential) weights of the fuzzy indices. Since the procedure is reversible, it was easy to check that the exponential weights (decision-maker preference) were correct.

- From a graphical point of view, the graph was checked with the two attributes calculated (investment cost and emissions) with respect to the attributes analyzed and the intrinsic cost. The results are logical for the two scenarios (Argentine production chain and society). The costs per MWh obtained are in accordance with those established by the Argentine Renewable Energy Chamber and the Ministry of Energy (Argentina, 2022): renewable projects cost around $1.5\ USD/MWh$. The global values of IC are a minimum of $18\frac{USD}{t\ CO2\ eq}$ and a maximum of $110\frac{USD}{t\ CO2\ eq}$ and, since 2020, the historical values of carbon bond prices are a minimum of $20\frac{USD}{t\ CO2\ eq}$ and a maximum of $100\frac{USD}{t\ CO2\ eq}$, so the prices obtained are in line with international values. Thus, the values obtained in the indices are consistent with national and international studies, according to the following references (Camargo, 2022b; Argentina, 2022; Investing.com, 2022). If the efficiency index (tp) increases, higher investments are required to obtain the same resulting emission. Secondly, the IC is found to be associated with the slope of the curve of the analyzed attributes, for constant efficiency (Ceteris Paribus) and an analyzed t-norm. The IC is a good indicator of energy efficiency and sustainability, as it encompasses the economic cost and the environmental cost or benefit (CO_2 emissions). The improvement of the IC index allowed the definition to be successfully extended to other types of t-norms (continuous and differentiable) in this work. Secondly, the most demanding operator are the Einstein's Product (EP) and Particular Hamacher's Product (PHP).

- In both scenarios (production chain and society), the EP is the most demanding with respect to the attribute curve analyzed (Section 3.5). In addition, Einstein's product has the most uniform intrinsic cost, which would be an advantage to avoid excessive prices and high penalties. In the case that the regulator wants to reduce CO_2 emissions by increasing the cost of the investment, it will have a better economic valuation with a greater decrease in emissions as the cost increases (slope of the curve).

- When analyzing the IC curves, and also when the expressions obtained in the mathematical development are compared, there is an "inflection point" given by the equality in the fuzzy indices. At this point, all t-norms agree on IC. The intrinsic cost at which all t-norms are equalized, at equal fuzzy indices in $i=1$ (Emissions) and $j=2$

(Investment Cost), is almost $40\frac{USD}{t\ CO2\ eq}$. For values lower than this reference value, the EP is the most demanding and the PHP is the least demanding with respect to the IC. For high values of Emissions, this ranking is reversed.

- This result has an impact on the decision-maker's intended use of the intrinsic cost. From society's point of view, if high emissions are desired, it is advantageous to use the EP to have minimum penalties and taxes. Also, if low emissions are desired, it is advantageous to use the EP to have low penalties in those cases, although they will be the highest in the case of high emissions. Therefore, if high stringency or incentives are required in the index regulation mechanisms, then the "best alternative" to use is the EP. However, this fuzzy operator and the IC are the most difficult to calculate.
- Thus, there is no optimal t-norm, but rather a "most satisfactory" one, which depends on the intended application of the proposed index. This shows a great versatility and practical application of the index to obtain the carbon price.

Then, the results are coherent, logical, satisfactory and promising. They are possible to apply in other areas of the productive sector, using other interest indices.

Further comparisons will be made in future work (including the influence of exponential weights) and mechanisms based on the Computable General Equilibrium (CGE) model and this methodology will be carried out to determine the carbon market price of the production chain. In addition, the procedure will be applied to determine the price of supplied and non-supplied energy investment projects based on these proposed indices. In addition, mechanisms for calculating the economic amount of the penalty or subsidy for the externality produced will be carried out. In this work, innovative environmental assessment mechanisms have been developed. With this carbon price, the regulator will provide an economic incentive for the company to make the relevant investments accordingly and the constant search for energy efficiency.

References

Amenta, P., Lucadamo, A., & Marcarelli, G. (2021). On the choice of weights for aggregating judgments in non-negotiable AHP group decision making. *European Journal of Operational Research*, *288*(1), 294-301. http://dx.doi.org/10.1016/j.ejor.2020.05.048.

Argentina. Ministerio de Economia. Secretaría de Energía. (2022). *Datos Energía*. Ministerio de Energía y Minería. Retrieved in 2022, October 12, from http://datos.energia.gob.ar/

Aydin, C., & Esen, Ö. (2018). Reducing CO2 emissions in the EU member states: Do environmental taxes work? *Journal of Environmental Planning and Management*, *61*(13), 2396-2420. http://dx.doi.org/10.1080/09640568.2017.1395731.

Camargo, F. G., & Schweickardt, G. A. (2014). Estimación de la tasa de retorno energético: análisis comparativo de las metodologías disponibles en la actualidad. *MASKANA, I+D+ingeniería*, 65-73. Retrieved in 2022, October 12, from https://publicaciones.ucuenca.edu.ec/ojs/index.php/maskana/article/view/575

Camargo, F. G. (2021). Survey and calculation of the energy potential and solar, wind and biomass EROI: application to a case study in Argentina. *Dyna*, *88*(219), 50-58. http://dx.doi.org/10.15446/dyna.v88n219.95569.

Camargo, F. G. (2022a). Dynamic modeling of the energy returned on invested. *Dyna*, *89*(221), 50-59. http://dx.doi.org/10.15446/dyna.v89n221.97965.

Camargo, F. G. (2022b). Fuzzy multi-objective optimization of the energy transition towards renewable energies with a mixed methodology. *Production*, *32*, e20210132. http://dx.doi.org/10.1590/0103-6513.20210132.

Camargo, F. G., Schweickardt, G. A., & Casanova, C. A. (2018). Maps of Intrinsic Cost (IC) in reliability problems of medium voltage power distribution systems through a Fuzzy multi-objective model. *Dyna*, *85*(204), 334-343. http://dx.doi.org/10.15446/dyna.v85n204.65836.

Camargo, F. G., Casanova Pietroboni, C. A., Pérez, E., & Schweickardt, G. A. (2019). Metodología regulatoria para propiciar la eficiencia energética desde el lado de la oferta con penetración de fuentes primarias de energías renovables. Parte 1: Descripción y alcance del modelo de optimización. *Investigacion Operativa*, *27*(45), 5-24. Retrieved in 2022, October 12, from https://ria.utn.edu.ar/handle/20.500.12272/5349

Camargo, F. G. (2019). *Metodología regulatoria para propiciar la eficiencia energética desde el lado de la oferta en sistemas de distribución de energía eléctrica* [Tesis doctoral]. Universidad Tecnológica Nacional, Santa Fe. Retrieved in 2022, October 12, from http://hdl.handle.net/20.500.12272/7010

Cavallaro, F., Danielis, R., Nocera, S., & Rotaris, L. (2018). Should BEVs be subsidized or taxed? A European perspective based on the economic value of CO2 emissions. *Transportation Research Part D, Transport and Environment*, *64*, 70-89. http://dx.doi.org/10.1016/j.trd.2017.07.017.

Hao, Y., Tian, C., & Wu, C. (2020). Modelling of carbon price in two real carbon trading markets. *Journal of Cleaner Production*, *244*, 118556. http://dx.doi.org/10.1016/j.jclepro.2019.118556.

Ichihashi, S. (2021). The economics of data externalities. *Journal of Economic Theory*, *196*, 105316. http://dx.doi.org/10.1016/j.jet.2021.105316.

Investing.com. (2022). *Materias Primas. Futuros emisiones de carbono*. Ministerio de Energía y Minería. Retrieved in 2022, October 12, from https://es.investing.com/commodities/carbon-emissions-historical-data

Janiesch, C., Zschech, P., & Heinrich, K. (2021). Machine learning and deep learning. *Electronic Markets*, *31*(3), 685-695. http://dx.doi.org/10.1007/s12525-021-00475-2.

Jeanne, O., & Korinek, A. (2019). Managing credit booms and busts: A Pigouvian taxation approach. *Journal of Monetary Economics*, *107*, 2-17. http://dx.doi.org/10.1016/j.jmoneco.2018.12.005.

Li, Z., Dai, H., Sun, L., Xie, Y., Liu, Z., Wang, P., & Yabar, H. (2018). Exploring the impacts of regional unbalanced carbon tax on CO2 emissions and industrial competitiveness in Liaoning province of China. *Energy Policy*, *113*, 9-19. http://dx.doi.org/10.1016/j.enpol.2017.10.048.

Lin, B., & Jia, Z. (2019). How does tax system on energy industries affect energy demand, CO2 emissions, and economy in China? *Energy Economics*, *84*, 104496. http://dx.doi.org/10.1016/j.eneco.2019.104496.

Liu, Y., Eckert, C. M., & Earl, C. (2020). A review of fuzzy AHP methods for decision-making with subjective judgements. *Expert Systems with Applications*, *161*, 113738. http://dx.doi.org/10.1016/j.eswa.2020.113738.

Maamoun, N. (2019). The Kyoto protocol: empirical evidence of a hidden success. *Journal of Environmental Economics and Management*, *95*, 227-256. http://dx.doi.org/10.1016/j.jeem.2019.04.001.

Miyamoto, M., & Takeuchi, K. (2019). Climate agreement and technology diffusion: Impact of the Kyoto Protocol on international patent applications for renewable energy technologies. *Energy Policy*, *129*, 1331-1338. http://dx.doi.org/10.1016/j.enpol.2019.02.053.

Pradhan, B. K., & Ghosh, J. (2022). A computable general equilibrium (CGE) assessment of technological progress and carbon pricing in India's green energy transition via furthering its renewable capacity. *Energy Economics*, *106*, 105788. http://dx.doi.org/10.1016/j.eneco.2021.105788.

Robson, E. N., Wijayaratna, K. P., & Dixit, V. V. (2018). A review of computable general equilibrium models for transport and their applications in appraisal. *Transportation Research Part A, Policy and Practice*, *116*, 31-53. http://dx.doi.org/10.1016/j.tra.2018.06.003.

Saaty, T. L. (2003). Decision-making with the AHP: why is the principal eigenvector necessary. *European Journal of Operational Research*, *145*(1), 85-91. http://dx.doi.org/10.1016/S0377-2217(02)00227-8.

Schweickardt, G., & Miranda, V. (2009). A two-stage planning and control model toward economically adapted power distribution systems using analytical hierarchy processes and fuzzy optimization. *International Journal of Electrical Power & Energy Systems*, *31*(6), 277-284. http://dx.doi.org/10.1016/j.ijepes.2009.03.003.

Schweickardt, G., & Pistonesi, H. (2010). Un modelo posibilístico para estimar el costo intrínseco de la energía no suministrada en sistemas de distribución eléctrica. *Dyna*, *77*(162), 249-259.

Shahzadi, G., Akram, M., & Al-Kenani, A. N. (2020). Decision-making approach under Pythagorean fuzzy Yager weighted operators. *Mathematics*, *8*(1), 70. http://dx.doi.org/10.3390/math8010070.

Silajdzic, S., & Mehic, E. (2018). Do environmental taxes pay off? The impact of energy and transport taxes on CO2 emissions in transition economies. *South East European Journal of Economics and Business*, *13*(2), 126-143. http://dx.doi.org/10.2478/jeb-2018-0016.

Simshauser, P. (2018). Price discrimination and the modes of failure in deregulated retail electricity markets. *Energy Economics*, *75*, 54-70. http://dx.doi.org/10.1016/j.eneco.2018.08.007.

Song, Y., Liu, T., Liang, D., Li, Y., & Song, X. (2019). A fuzzy stochastic model for carbon price prediction under the effect of demand-related policy in China's carbon market. *Ecological Economics*, *157*, 253-265. http://dx.doi.org/10.1016/j.ecolecon.2018.10.001.

Sun, W., & Huang, C. (2020). A carbon price prediction model based on secondary decomposition algorithm and optimized back propagation neural network. *Journal of Cleaner Production*, *243*, 118671. http://dx.doi.org/10.1016/j.jclepro.2019.118671.

Sun, W., & Zhang, C. (2018). Analysis and forecasting of the carbon price using multi—resolution singular value decomposition and extreme learning machine optimized by adaptive whale optimization algorithm. *Applied Energy*, *231*, 1354-1371. http://dx.doi.org/10.1016/j.apenergy.2018.09.118.

Tsao, Y.-C., Nugraha Ridhwan Amir, E., Thanh, V.-V., & Dachyar, M. (2021). Designing an eco-efficient supply chain network considering carbon trade and trade-credit: a robust fuzzy optimization approach. *Computers & Industrial Engineering*, *160*, 107595. http://dx.doi.org/10.1016/j.cie.2021.107595.

Wei, S., Chongchong, Z., & Cuiping, S. (2018). Carbon pricing prediction based on wavelet transform and K-ELM optimized by bat optimization algorithm in China ETS: the case of Shanghai and Hubei carbon markets. *Carbon Management*, *9*(6), 605-617. http://dx.doi.org/10.1080/17583004.2018.1522095.

Zhang, J., & Zhang, Y. (2018). Carbon tax, tourism CO2 emissions and economic welfare. *Annals of Tourism Research*, *69*, 18-30. http://dx.doi.org/10.1016/j.annals.2017.12.009.

Zhao, L. T., Miao, J., Qu, S., & Chen, X. H. (2021). A multi-factor integrated model for carbon price forecasting: market interaction promoting carbon emission reduction. *The Science of the Total Environment*, *796*, 149110. http://dx.doi.org/10.1016/j.scitotenv.2021.149110. PMid:34328877.

Annex 1. T-norm product comparison.

The relative valuation of the t-norm requirement depends on the values of the preference functions (Table 2). The absolute value of the intrinsic cost obtained with the Einstein product, the algebraic product, the particular Hamacher product and the generic Hamacher product are presented in Equation 17, Equation 21, Equation 26 and Equation 32, respectively. Dividing the IC functions of each t-norm (in a matrix similar to the preference matrix) gives Table 2.

Table 2. Intrinsic cost ratio for the different t-norms: Algebraic Product - AP, Particular Hamacher's Product - PHP (with null p), General Hamacher's Product - GHP and Einstein's Product – EP.

	AP	PHP	GHP	EP
AP	1	$\frac{\mu_i}{\mu_j}$	$\frac{p-\mu_i\bullet(p-1)}{p-\mu_j\bullet(p-1)}$	$\frac{\mu_i-2}{\mu_j-2}$
PHP	$\frac{\mu_j}{\mu_i}$	1	$\left(\frac{\mu_j}{\mu_i}\right)\left(\frac{p-\mu_i\bullet(p-1)}{p-\mu_j\bullet(p-1)}\right)$	$\left(\frac{\mu_j}{\mu_i}\right)\left(\frac{\mu_i-2}{\mu_j-2}\right)$
GHP	$\frac{p-\mu_j\bullet(p-1)}{p-\mu_i\bullet(p-1)}$	$\left(\frac{\mu_i}{\mu_j}\right)\left(\frac{p-\mu_i\bullet(p-1)}{p-\mu_j\bullet(p-1)}\right)$	1	$\left(\frac{p-\mu_j\bullet(p-1)}{p-\mu_i\bullet(p-1)}\right)\left(\frac{\mu_i-2}{\mu_j-2}\right)$
EP	$\frac{\mu_j-2}{\mu_i-2}$	$\left(\frac{\mu_i}{\mu_j}\right)\left(\frac{\mu_j-2}{\mu_i-2}\right)$	$\left(\frac{p-\mu_j\bullet(p-1)}{p-\mu_i\bullet(p-1)}\right)\left(\frac{\mu_i-2}{\mu_j-2}\right)$	1

Source: The Author.

The numerator is associated with the t-norm of the row and the denominator with the t-norm of the column. In the columns, the Algebraic Product - AP, the Particular Hamacher Product - PHP, the General Hamacher Product - GHP and the Einstein Product - EP are represented in the numerators. In the rows, the intrinsic costs of the Algebraic Product - AP, the Particular Hamacher Product - PHP, the General Hamacher Product - GHP and the Einstein Product - EP are represented in the denominators.

When the preference functions of attributes $i=1$ (Emissions) and $j=2$ (investment cost) are equal, then all t-norms have the same requirement regarding intrinsic cost. When the preference function associated with j is greater than the preference function associated with i, the tabulated values are greater than the unity. This indicates that the intrinsic costs associated to the rows are higher than the intrinsic cost of the corresponding column. This means that the t-norm associated with the denominator is more demanding than the t-norm associated with the denominator and vice versa for the opposite case.

Furthermore, the value of the preference function depends on the exponential weights used. It is observed that the largest difference between the evaluations is associated with the c and d component (Equation 9). This is associated with the subjective component of the index. The consequence of this is that the values will be different, obtaining a scale of t-norm operators, in terms of requirements. This is interesting for determining incentive and sanction mechanisms for the regulatory authority with different stringency criteria, depending on the needs. Comparisons of these expressions are made in the annex, determining the level of stringency.

It is observed that if $\mu_i = \mu_j$, then all the values in the table will be unitary and therefore the intrinsic costs will be equal. This point corresponds to the "inflection point" of Figure 10, where the order of requirement of the t-norm changes.

This analysis is similar (not the same) to the construction of preference matrices in AHP. This Annex 1. aims to show more clearly why the t-norm curves intersect at the same point. The AHP method used in this paper is explained in section 2.1. For more information on AHP methods, the following references are presented (Saaty, 2003; Schweickardt & Miranda, 2009; Camargo, 2019; Liu et al., 2020).